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COLOUR THRESHOLDS DEFINITION BASED ON THE STATISTICAL APPROACH

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ABSTRACT

The authors developed the MatLab program for modelling the observation of an object and a background of different chromaticity at the constant luminance and obtained the uniform-chromaticity curves with shape close to elliptical. The results of colour thresholds calculations are compared with MacAdam ellipses and an explanation is given for their differences. Based on the statistical model of threshold colour vision, the authors derived expressions for the physiological uniform colour space.

Keywords: colour-matching coefficients, colour-matching functions *(CMF)*, dichromats, trichromats, statistical model of threshold colour vision, threshold characteristics of vision, object detection, MacAdam ellipses, uniform colour space (UCS)

1. INTRODUCTION

Currently TC CIE90 is creating a roadmap for the development of a new, complete, self-consistent system of CIE colorimetry measures based directly on cone fundamentals [1]. The ***XYZ*** functions are linear combinations of the actual cone fundamentals ***LMS***. However, the proposed linear combinations still do not satisfactorily match the ***XYZ*** colorimetric system and the cone fundamentals ***LMS*** obtained from dichromate studies.

The sum of the positive tristimulus values of the initial ***LMS*** system shall also have a positive value of the ***Y*** coordinate after transformation into ***XYZ*** since only this coordinate estimates the total level of receptor excitement of a trichromat [2].

This issue is especially relevant for the field of colour differences research, where uniform colour spaces (UCS) based on the theory of colour vision are proposed [3,4].

2. THE METHOD OF DEFINITION OF COLOUR THRESHOLDS

With the statistical threshold theory of colour vision the authors got the expression differences between spectral distributions of radiance $L_{\text{co}}^{th.dif}(\lambda)$ of an object and a background for monochromatic radiations at the visual spectrum wavelengths allowing threshold detection of the objects by an observer [5]:

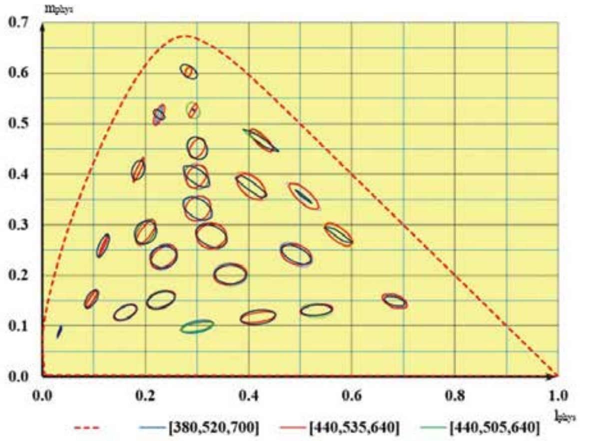

Fig. 1 The results of calculation of colour thresholds $(\boldsymbol{LMS})_{phys}$ multiplied by 10 times

$$L_{eo}^{th.dif}(\lambda)=\frac{C_1}{\sqrt{C_2\left(\dfrac{\bar{l}(\lambda)}{\int\limits_{380}^{780}L_{e\lambda b}(\lambda)\bar{l}(\lambda)d\lambda}\right)^2+C_3\left(\dfrac{\bar{m}(\lambda)}{\int\limits_{380}^{780}L_{e\lambda b}(\lambda)\bar{m}(\lambda)d\lambda}\right)^2+\left(\dfrac{\bar{s}(\lambda)}{\int\limits_{380}^{780}L_{e\lambda b}(\lambda)\bar{s}(\lambda)d\lambda}\right)^2}}, \quad (1)$$

where $C_1 - C_3$ are the constant coefficients, $L_{e\lambda b}(\lambda)$ is the spectral distributions of radiance of a white background.

To calculate the threshold chromaticity differences of a rectangular object with an angular size of 2 degrees at a background with constant luminance 10 cd/m^2, a Matlab program was developed. The program simulated the observation of an object and a background of different chromaticity and the constant luminance. The necessary luminance and the chromaticity of the object and the background were created by three monochromatic light sources. Calculated results were obtained for three variants of monochromatic light sources: with wavelength 380 nm, 520 nm, 700 nm; with λ equal to 440 nm, 505 nm, 640 nm, and with λ equal to 440 nm, 535 nm, 640 nm.

Threshold dependences for tristimulus values were calculated in the ***LMS****phys* system, Fig. 1, then the results obtained were converted to the ***XYZ*** system using the transition matrix (2), after which the threshold chromaticity coordinates were calculated in this coordinate system, Fig. 2:

$$\left(\bar{x}(\lambda)\;\bar{y}(\lambda)\;\bar{z}(\lambda)\right)=\left(\bar{l}_{phys}(\lambda)\bar{m}_{phys}(\lambda)\bar{s}_{phys}(\lambda)\right)\times$$
$$\times\begin{pmatrix}1.261118115 & 0.455469536 & 0.000000000\\ -0.464970813 & 0.543209593 & 0.000000000\\ 0.203852698 & 0.001320870 & 1.000000000\end{pmatrix}. \quad (2)$$

Uniform-chromaticity curves with shape close to elliptical depend not only on the chromaticity coordinates but also on the spectral distribution of radiation that provides these colours.

With changes of radiation spectrum, not only the size of the ellipses but also their orientation in the colour space changes, Fig. 2.

3. THE DEVELOPMENT OF A NEW UNIFORM COLOUR SPACE

In the article [6], the expression for the threshold conditions of observation is derived:

$$m_\Lambda = m_l^n + m_m^n + m_s^n = \ln\Lambda_{th}, \quad (3)$$

where m_l^n, m_m^n, m_s^n – mathematical expectation of the natural logarithm of the likelihood ratio (Λ) and $\ln\Lambda_{th}$ is the natural logarithm of the threshold value of the likelihood ratio.

For above threshold conditions:

$$m_l + m_m + m_s = n\ln\Lambda_{th}, \quad (4)$$

where n is the number of threshold values $\ln\Lambda_{th}$ that is contained in $\ln\Lambda$.

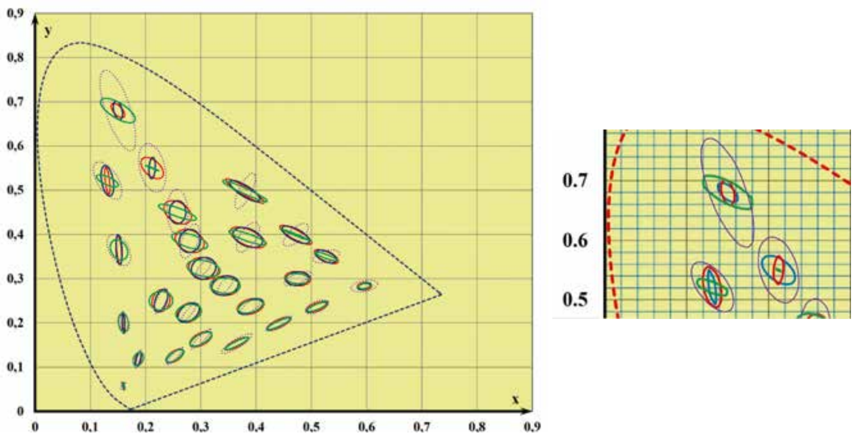

Fig. 2. Results of colour thresholds calculations and experimental MacAdam ellipses multiplied by 10 times

The authors divide the left and the right sides of the equation (4) by $\left(1-\frac{m_s}{n\ln\Lambda_{th}}\right)\ln\Lambda_{th}$:

$$\frac{\frac{m_l}{\ln\Lambda_{th}}}{\left(1-\frac{m_s}{n\ln\Lambda_{th}}\right)}+\frac{\frac{m_m}{\ln\Lambda_{th}}}{\left(1-\frac{m_s}{n\ln\Lambda_{th}}\right)}=n. \quad (5)$$

Taking it (4) into consideration, the authors get:

$$\frac{\frac{m_l}{\ln\Lambda_{th}}}{\left(1-\frac{m_s}{m_l+m_m+m_s}\right)}+\frac{\frac{m_m}{\ln\Lambda_{th}}}{\left(1-\frac{m_s}{m_l+m_m+m_s}\right)}=n. \quad (5a)$$

Then the authors get:

$$M_l^2+M_m^2=\left(\sqrt{n}\right)^2, \quad (5b)$$

where

$$M_l=\frac{\sqrt{\frac{m_l}{\ln\Lambda_{th}}}}{\sqrt{1-\frac{m_s}{m_l+m_m+m_s}}}\rightarrow M_l=\frac{\sqrt{\frac{m_l}{m_l^i+m_m^i+m_s^i}}}{\sqrt{1-\frac{m_s}{m_l+m_m+m_s}}}, \quad (5c)$$

$$M_m=\frac{\sqrt{\frac{m_m}{\ln\Lambda_{th}}}}{\sqrt{1-\frac{m_s}{m_l+m_m+m_s}}}\rightarrow M_m=\frac{\sqrt{\frac{m_m}{m_l^i+m_m^i+m_s^i}}}{\sqrt{1-\frac{m_s}{m_l+m_m+m_s}}}.$$

Thus, the measure of the colour difference between the reference and the studied radiation is the distance from the origin (the point of luminance and chromaticity of the reference) to the point with coordinates [$\boldsymbol{M_l}$, $\boldsymbol{M_m}$] equal to $\sqrt{M_l^2+M_m^2}$.

Fig. 3 illustrates the [$\boldsymbol{M_l}$, $\boldsymbol{M_m}$] chromaticity diagram, where all points of threshold ellipses are transformed into points on one circle with a unit radius. Different chromaticity corresponds to different locations of threshold points on this circle.

The proposed UCS differs from the known ones in that it does not display the colour itself, but the difference between the colours of two objects.

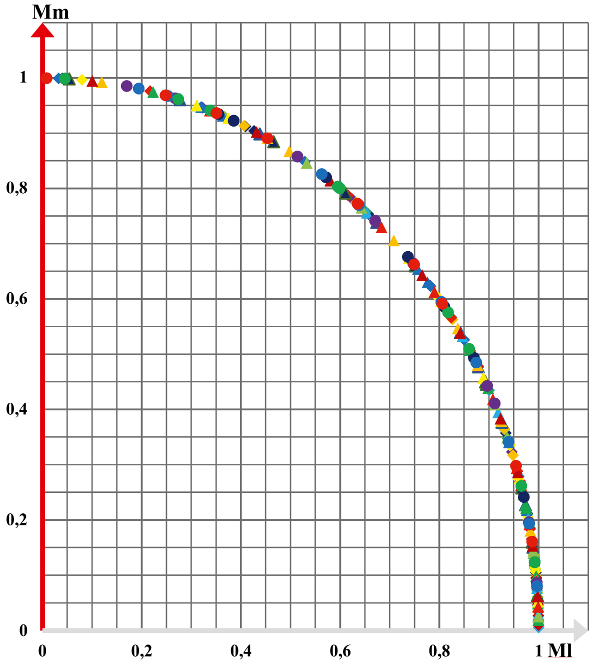

Fig. 3. The ratio between $\boldsymbol{M_l}$ and $\boldsymbol{M_m}$ coordinates changes, but the threshold value remains the same

4. DISCUSSION OF THE RESULTS

The obtained results show that when determining the threshold colour differences, it is necessary to take into account not only the colour of the object and the background, but also the spectrum of the radiation sources that create them.

The mathematical expectation m_Λ is determined by dependence (6), i.e. the sum of the squares of the ratios of integrals that determine the colour of the study, and the chromaticity coordinates and their difference are determined by another, fractional-linear dependence on the same integrals (7).

$$m_\Lambda=2\omega\left\{\begin{array}{l} C_{1l}\left(\frac{\int_{380}^{780}\Delta L_{e\lambda o}(\lambda)\bar{l}(\lambda)d\lambda}{\int_{380}^{780}L_{e\lambda b}(\lambda)\bar{l}(\lambda)d\lambda}\right)^2+ \\ +C_{1m}\left(\frac{\int_{380}^{780}\Delta L_{e\lambda o}(\lambda)\bar{m}(\lambda)d\lambda}{\int_{380}^{780}L_{e\lambda b}(\lambda)\bar{m}(\lambda)d\lambda}\right)^2+ \\ +C_{1s}\left(\frac{\int_{380}^{780}\Delta L_{e\lambda o}(\lambda)\bar{s}(\lambda)d\lambda}{\int_{380}^{780}L_{e\lambda b}(\lambda)\bar{s}(\lambda)d\lambda}\right)^2 \end{array}\right\}, \quad (6)$$

$$\Delta\xi = \frac{\int_{380}^{780} L_{e\lambda o}(\lambda)\bar{\xi}(\lambda)d\lambda}{\int_{380}^{780} L_{e\lambda o}(\lambda)\bar{l}(\lambda)d\lambda + \int_{380}^{780} L_{e\lambda o}(\lambda)\bar{m}(\lambda)d\lambda + \int_{380}^{780} L_{e\lambda o}(\lambda)\bar{s}(\lambda)d\lambda} - \frac{\int_{380}^{780} L_{e\lambda b}(\lambda)\bar{\xi}(\lambda)d\lambda}{\int_{380}^{780} L_{e\lambda b}(\lambda)\bar{l}(\lambda)d\lambda + \int_{380}^{780} L_{e\lambda b}(\lambda)\bar{m}(\lambda)d\lambda + \int_{380}^{780} L_{e\lambda b}(\lambda)\bar{s}(\lambda)d\lambda}, \quad (7)$$

where $\xi = \{l, m, s\}$.

The construction of generally accepted UCS (such as ***CIELuv***, ***CIELab***) is based on a non-linear transformation of the usual ***XYZ*** colorimetric colour space. The main disadvantage of this approach is the formal nature of the transformations, which does not reflect the essence of physiological processes. Therefore, all systems based on graphical transformations of the CIE chromaticity diagram are valid only for those conditions in which the MacAdam ellipses were measured. As is known, these data were based on only one observer, and all stimuli were at the same luminance level.

The more the ***LMS*** reference colour stimuli differ from reference colour stimuli of the physiological system of observers on the basis of which the ***XYZ*** system was built, and, consequently, the colour-matching functions differ from the curves of the spectral sensitivity of the eye receptors, the greater the error in the nonlinear transformation and further extrapolation.

The main advantages of the proposed UCS are the derivation of its coordinates from the statistical model of threshold colour vision, rather than obtaining it in the traditional way by selecting coefficients in nonlinear transformations. After the determining of the M_l and M_m coordinates, the colour space becomes UCS and does not depend on the chromaticity and spectral distribution of the radiations.

REFERENCES

1. CIE 1-98: A Roadmap Toward Basing CIE Colorimetry on Cone Fundamentals (https://cie.co.at/technicalcommittees/roadmap-toward-basing-cie-colorimetry-cone-fundamentals)

2. Boos, G. V., Grigoryev, A. A., Rybina, V.A. Research into Monochromatic Colour Vision Thresholds and Determination of the Trichromatic Colour-Matching Coefficients // Light & Engineering Journal, 2022, Vol. 30, # 2, pp. 89–19.

3. Meshkov, V.V., Matveev, A.B. Basics of Light Engineering: Study Guide for Higher Education Institutions: in 2 parts. P. 2. Physiological Optics and Colorimetry [Osnovy svetotekhniki: uchebnoye posobiye dlya vuzov: v 2 ch. Ch. 2. Fiziologicheskaya optika i kolorimetriya] 2nd Edition, revised and supplemented, Moscow: Energoatomizdat, 1989, 432 p.

4. Yasuhisa Nakano, Naoki Obayashi, Kenichiro Suehara, Jiro Kohda, Takuo Y. Integration of uniform colour space and colour appearance model // AIC COLOR2002 SI Color & Textiles Proceedings, 2002, pp. 128–133.

5. Boos, G. V., Grigoryev, A. A., Rybina, V.A. The Experimental Research Installation for the Determination of Monochromatic Thresholds of Human Visual System Vision // Light & Engineering Journal, 2021, Vol. 29, # 6, pp. 34–42.

6. Boos, G.V. The Probability of Detecting Coloured Objects on Coloured Backgrounds Based on a Statistical Model of the Threshold of Colour Vision // Light & Engineering Journal, 2018, Vol. 26, # 2, pp. 14–19.

7. MacAdam, D.L. Visual Sensitivities to Colour Differences in Daylight // J. Opt. Soc. Am., 1942, Vol. 32, pp. 247–274.

8. Brown, W.R.L., MacAdam, D.L. Visual sensitivities to combined chromaticity and luminance differences // J. Opt. Soc. Am., 1949, Vol. 39, # 10, pp. 808–834.

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