

RESEARCH INTO MONOCHROMATIC COLOUR VISION THRESHOLDS AND DETERMINATION OF THE TRICHROMATIC COLOUR-MATCHING COEFFICIENTS

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ABSTRACT

The authors experimentally defined the dependences of colour thresholds between an object and a background for 10 trichromatic observers. The experiments included observation of monochromatic objects with dimension of (2×2) angular degree against a white background. Based on the statistical theory of human threshold colour vision, it was proved that in the XYZ colorimetric system, the value of Y coordinate of the primary colour M_{phys} is positive and lies along the $y=1-x$ line. This fact allowed to define all the characteristics of a trichromatic physiological system $(LMS)_{phys}$ necessary for chromatic calculations.

Keywords: colour-matching functions, dichromate, trichromate, statistic theory, threshold characteristics of vision, object detection, experimental studies installation, MacAdam ellipses

1. INTRODUCTION

Light engineers have been actively studying chromatic characteristics of spectators with normal colour vision since the second half of the 18th century [1, 2] and in the 20th century [3–6]. These works allowed the International Commission on Illumination (CIE) to develop and to adopt such colorimetric systems as RGB , XYZ (1931) [7] and $X_{10}Y_{10}Z_{10}$ (1964) for chromatic calculations [8]. XYZ and $X_{10}Y_{10}Z_{10}$ colorimetric systems are convenient for chromatic calculations but their co-

lour-matching functions were obtained for primary colours not related to the primary colours of the normal trichromatic human vision system. This does not allow accounting for a number of colour perception distinctions in calculations, in particular, chromatic adaptation of the vision system to radiations of different chromaticity. Either colour-matching functions or the functions of spectral responsivity of the trichromatic vision system are required to account for this phenomenon. Direct experimental determination of these functions is impossible since even monochromatic radiations of the visual spectrum are responded by two receptors of the vision system simultaneously. Such feature of a trichromatic vision system makes it necessary to use indirect methods to define their colour-matching functions.

One of the first methods, proposed by the developer of the colour space theory D.C. Maxwell [1], is the method based on studying of dichromate, i.e. people without sensitivity of one of the three types of light-sensitive receptors: R , G or B (LMS in English-language literature). It was shown in [2] that in any colorimetric system, a pencil of lines for chromaticity not perceived by dichromate intersects in a point with the tristimulus values of the primary colour that a dichromate is not sensitive to. The main condition to be met in this case is equality of spectral responsivities of the remaining receptors of dichromate to corresponding responsivities of trichromatic receptors. This assumption is also used in contemporary approaches to definition of

Table 1. Experimental Dependences $(L_{co}^{th.dif}(\lambda))^{-1}$ [sr · m²/W] for 10 Trichromatic Vision Observers

λ , nm	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	$(L_{co}^{th.dif})^{-1}$	RMS / $L_{co}^{th.dif}$, %
420	238	318	217	194	272	455	193	220	190	159	246	35.2
435	703	881	1258	1808	1750	1293	707	1489	1171	1108	1217	32.0
450	823	1105	1292	2386	2119	1543	661	1794	1517	1334	1457	37.1
465	519	998	1004	1357	1893	996	487	1244	1126	1058	1068	37.6
480	495	797	548	673	911	438	300	641	635	578	602	29.0
495	1139	912	702	526	778	644	685	960	689	756	779	22.9
510	1900	1551	1582	1084	1368	1637	1628	1887	1964	1549	1615	16.4
525	2799	2442	2765	1975	2303	2415	2739	3056	3100	2265	2586	14.1
540	3691	3126	3088	2159	3218	3114	4084	4141	3683	3640	3394	17.2
555	3792	2920	2669	1983	3293	3367	4303	4143	3836	3361	3367	21.0
570	3473	2275	2591	1679	2405	2411	4110	3232	3328	3094	2860	24.9
585	3328	2051	3357	1982	2009	2227	4232	1672	2889	4029	2777	33.1
600	2961	2413	2842	2487	2160	2934	3428	966	2959	3767	2692	28.5
615	2186	1752	2485	2242	1829	2776	1512	865	2336	2775	2076	28.7
630	1094	1648	1933	1979	1617	1943	609	503	1092	1852	1427	39.3
645	388	1233	1047	1075	1088	1066	174	368	362	832	763	51.8
660	104	981	550	614	672	437	26	171	80	307	394	79.0

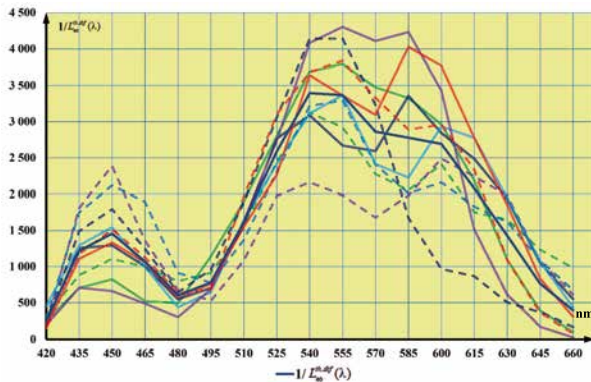


Fig. 1. Experimental dependences $(L_{co}^{th.dif}(\lambda))^{-1}$ [sr · m²/W] for 10 trichromatic vision observers and their average values

responsivities of trichromatic vision based on isolation of one of the types of dichromate cones [9]. As we noted previously [10], there are still doubts about legitimacy of the approach involving dichromate rather than trichromatic case even if they are selected, based on DNA tests. This is due to the fact that colour mixture curves of the human vision system (VS) are defined not only by cone responsivity. Human brightness and chromaticity perceptions appear not at receptor outputs but after their signals are processed by the brain and this algorithm (yet unknown) may be completely different in the cases of trichromatic vision person, who process signals from three receptors, and dichromatic, i.e. without

one of the receptors, but still perceiving colours. These doubts are the grounds of the necessity to define spectral responsivity of a trichromatic vision system without involving dichromate, which is **the main goal of this work**. This article describes such method and the experimental results obtained.

In [10], the authors explained the concept of a physiological chromatic system $(RGB)_{phys}$ for English-speaking readers. To understand non-ambiguously which systems are mentioned in this article, the following designations are used:

- The CIEPO06 model or the *LMS* system with the functions $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, $\bar{s}(\lambda)$, so-called cone fundamentals;
- N.D. Nyberg’s and E.N. Yustova’s physiological chromatic system $(RGB)_{phys}$: $(LMS)_{NY}$;
- The system developed by the authors will be denoted $(LMS)_{phys}$ with the colour-matching functions $\bar{l}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$ respectively.

2. THE FIRST STAGE OF THE STUDIES

2.1. The Method of Obtaining the Results

Based on the general expression [11] defining the probability to identify coloured objects against coloured backgrounds, [10] describes the method of determination of trichromatic specific tristimu-

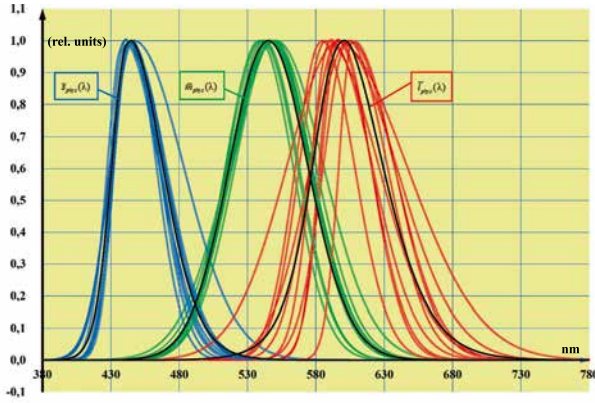


Fig. 2. Spectral dependences of colour-matching functions $(LMS)_{phys}$ per unit for 10 trichromats and their average values for colour-matching coefficients $\bar{L}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$

lus values based on the statistical threshold theory of colour vision as well as the experimental installation for application of this method. The method is based on solving of a system of equations non-linear in relation to the sought-after matching functions $\bar{L}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$ [10] by means of the least squares method [12]:

$$L_{co}^{th.dif}(\lambda) = \frac{C_1}{C_2 \left(\frac{\bar{L}_{phys}(\lambda)}{\int_{380}^{780} L_{e\lambda b}(\lambda) \bar{L}_{phys}(\lambda) d\lambda} \right)^2 + C_3 \left(\frac{\bar{m}_{phys}(\lambda)}{\int_{380}^{780} L_{e\lambda b}(\lambda) \bar{m}_{phys}(\lambda) d\lambda} \right)^2 + \left(\frac{\bar{s}_{phys}(\lambda)}{\int_{380}^{780} L_{e\lambda b}(\lambda) \bar{s}_{phys}(\lambda) d\lambda} \right)^2} \quad (1)$$

The left-hand side of the system contains the experimentally defined differences between spectral distributions of radiance $L_{co}^{th.dif}(\lambda)$ of an object and a background for monochromatic radiations at the visual-spectrum wavelengths allowing threshold detection of the objects by a spectator. The experimental studies were conducted using the installation described in [10] with constant luminance of the background equal to 11 cd/m² for monochromatic radiations with 17 different wavelength values.

10 trained spectators with normal colour vision checked by means of Rabkin’s polychromatic tables took part in the experiments. The method of minimal change of object appearance was used for determination of threshold contrast since it is the simplest one. Before the experiment, a series of preliminary experiments was conducted in order to develop a stable criterion of object detection in the spectators, which was controlled based on stabilisation of the threshold contrasts they obtained. The results of the experiments are presented in Table 1 and in Fig. 1.

The values of the threshold contrasts were measured within the range of (420–660) nm with increment of 15 nm for observations against a white background. The bandwidth of the monochromatic radiation of the observation object was 10 nm and its angular dimension was (2×2) degrees. Relative error of determination of the average value of $(L_{co}^{th.dif}(\lambda))^{-1}$ for 10 spectators and 200 demonstrations of the objects at each wavelength does not exceed 13.5 % (including the 7 % error of the experimental installation). Note the large scatter of threshold values of $(L_{co}^{th.dif}(\lambda))^{-1}$ for different spectators in relation to their average values (the last column of Table 1). Relative standard deviation at each wavelength for 10 spectators is equal to 30 %, which is explained by the natural scatter of their threshold characteristics.

When solving the system of 17 equations (1), the following approximations for values of $\bar{L}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$ were used [10]:

$$\bar{L}_{phys}(\lambda) = \frac{f_l(\lambda)}{2} \exp \left\{ - \left[\frac{(\lambda - \lambda_{lm})}{\sigma_l} \right]^2 \right\} + \frac{1 - f_l(\lambda)}{2} \exp \left\{ - \left[\frac{(\lambda - \lambda_{lm})}{C_l \sigma_l} \right]^2 \right\}, \quad (2)$$

$$\bar{m}_{phys}(\lambda) = \exp \left\{ - \left[\frac{(\lambda - \lambda_{mm})}{\sigma_m} \right]^2 \right\},$$

$$\bar{s}_{phys}(\lambda) = \frac{f_s(\lambda)}{2} \exp \left\{ - \left[\frac{(\lambda - \lambda_{sm})}{\sigma_s} \right]^2 \right\} + \frac{1 - f_s(\lambda)}{2} \exp \left\{ - \left[\frac{(\lambda - \lambda_{sm})}{C_s \sigma_s} \right]^2 \right\},$$

Table 2. Values of Approximation Coefficients for 10 Observers and Their Values for Average $L_{co}^{th.dif}$

Observer #/ parameter	λ_{lm}	λ_{mm}	λ_{sm}	σ_l	σ_m	σ_s	C_l	C_s
1	595.8	550.7	444.9	28.4	49.5	22.0	1.24	1.19
2	594.1	546.2	446.6	16.6	43.6	23.7	4.50	2.19
3	587.5	538.7	440.2	27.8	35.0	14.7	1.97	2.72
4	604.5	540.4	442.1	40.4	36.9	14.0	1.16	2.37
5	596.8	547.9	443.3	18.3	41.4	15.9	3.27	2.43
6	603.6	546.4	445.2	25.7	40.5	22.6	1.59	1.28
7	584.4	549.0	439.0	27.3	39.9	16.4	1.25	2.30
8	606.8	546.3	442.5	14.2	41.4	15.4	3.07	2.21
9	597.5	542.3	444.0	54.8	36.1	16.5	0.60	2.21
10	590.5	549.5	442.4	20.0	43.3	15.1	2.19	2.51
$L_{co}^{th.dif}$	593.84	548.03	443.11	23.53	43.14	17.11	2.01	2.11

Table 3. Relative Approximation Errors of the Dependences $(L_{co}^{th.dif}(\lambda))^{-1}$ When Solving the System (1) for 10 Observers

λ nm/ η %	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	η_{avg} %	η_{max} %
420	0.052	0.018	0.033	0.128	0.095	0.012	0.034	0.004	0.047	0.132	0.055	0.132
435	0.317	0.092	0.492	1.247	0.722	0.057	0.715	0.052	0.267	1.282	0.524	1.282
450	0.939	0.630	3.179	6.247	6.001	0.116	3.668	0.691	0.823	2.582	2.488	6.247
465	1.017	0.841	4.031	8.145	10.540	0.041	4.372	1.351	0.274	5.426	3.604	10.540
480	3.638	0.445	1.759	2.510	6.613	0.183	2.962	0.994	1.394	2.802	2.330	6.613
495	6.212	0.429	0.761	0.831	4.292	4.078	2.197	4.078	6.766	1.771	3.142	6.766
510	1.550	1.286	1.637	3.927	3.055	9.033	1.728	3.211	10.595	2.286	3.831	10.595
525	3.590	0.022	2.457	5.507	3.978	4.760	5.813	5.511	0.755	11.395	4.379	11.395
540	1.757	3.172	1.787	3.269	2.443	4.908	2.877	0.797	8.247	9.013	3.827	9.013
555	0.081	1.502	0.369	0.189	3.915	4.811	1.483	2.862	1.034	1.922	1.817	4.811
570	0.047	1.280	0.000	0.863	2.453	1.414	3.352	6.153	4.902	0.004	2.047	6.153
585	0.293	0.990	3.152	1.578	0.136	0.630	1.265	5.206	3.592	0.020	1.686	5.206
600	1.777	10.615	8.005	2.175	0.645	1.100	8.215	2.199	0.823	1.710	3.726	10.615
615	2.697	9.812	0.936	4.290	4.210	0.872	11.948	10.151	3.877	3.672	5.247	11.948
630	0.055	0.617	8.628	7.772	5.569	0.503	2.775	14.083	2.265	6.455	4.872	14.083
645	2.611	4.977	2.171	7.827	0.215	0.659	9.500	4.407	1.106	0.460	3.393	9.500
660	1.296	3.749	1.423	2.450	1.350	0.279	4.076	0.039	0.952	1.953	1.756	4.076
η_{avg} %	1.643	2.381	2.401	3.468	3.308	1.968	3.940	3.635	2.807	3.111		

Table 2 summarises the values of the approximation coefficients for all 10 spectators and their evaluation for the average value of $L_{co}^{th.dif}(\lambda)$.

The average approximation error of the experimentally obtained values of $(L_{co}^{th.dif}(\lambda))^{-1}$ does not exceed 6 % and the maximum error for all spectators and experimental points is a little higher than 14 %. Table 3 summarises the approximation errors for all the studied observers.

Fig. 3 presents the initial (N1, N8, Average) and the approximated (N1_A, N8_A, Average_A) dependences $(L_{co}^{th.dif}(\lambda))^{-1}$ for the cases of the best (N1) and the worst approximation (N8) as well as for approximation of the average function $(L_{co}^{th.dif}(\lambda))^{-1}$ for 10 observers. The results demonstrate sufficient sensitivity of the method which allows us to identify the individual distinctions of colour visions in different spectators.

Table 4. The Matrix of Tristimulus Values Transformation from *LMS* to *XYZ*

	\bar{x}	\bar{y}	\bar{z}
\bar{l}	1.947354690	-1.414451230	0.364763270
\bar{m}	0.689902720	0.348321890	0
\bar{s}	0	0	1.934853430
Σ coordinate	2.637257410	-1.066129340	2.299616700

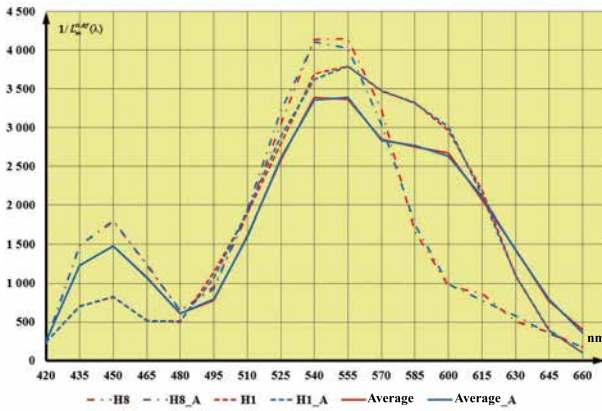


Fig. 3. Initial (red) and approximated (blue) dependences $(L_{co}^{th,dif}(\lambda))^{-1}$ (N1 is the lowest error, N8 is the largest error, solid curves are the average function $(L_{co}^{th,dif}(\lambda))^{-1}$ for 10 observers

2.2. Discussion of the Results Obtained at the First Stage of the Study

After defining the average spectral dependences $\bar{l}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$ the per-unit, values of which are shown by black colour in Fig. 2, it is necessary to define their correlation with the CIE standard: the colorimetric systems **RGB** and **XYZ** (1931). It is worth noting that we did not expect complete identity for the following reasons:

1. The average chromatic characteristics of the set of spectators we used for the experiments [3, 4] cannot be completely identical due to a finite number of spectators involved in the experiments.
2. The approximation expression we used (expressions (2)) may not allow to approximate average spectral dependences of the spectators [3, 4] with the necessary degree of accuracy since we do not know the distinctions of these dependences.

The analysis of the experimental results we obtained have shown that the matrices of transition from the colour systems of the studied spectators to the *XYZ* system provide a positive value of the *Y* coordinate of the primary colour M_{phys} on the chromaticity diagram and its tristimulus values lie along

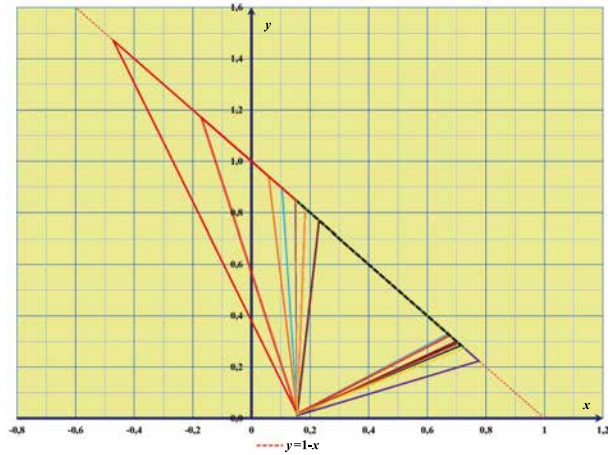


Fig. 4. Primary-colour triangles of the studied dichromate

the line $y = 1-x$ just as the coordinates of the L_{phys} colour. This result is obtained for all studied spectators and for characteristics of the averaged spectator. Fig. 4 demonstrates the reference colour stimuli (RCS) triangles of all the studied trichromatic vision systems in the *XYZ* system.

This results obtained by us when studying trichromatic vision system differ fundamentally from the results obtained when studying both classic dichromate [13] and dichromate selected based on DNA tests [9].

Fig. 5 demonstrates the RCS triangles of dichromate presented by Yustova [13] and obtained based on the matrix of transition from the *LMS* system to the *XYZ* system shown in [9]. The same figure also presents the *XYZ*, *LMS* and Yustova's *XYZ* chromaticity diagrams (dashed curves).

The negative value of the *M* coordinate for the dichromate chromatic system leads to the fact that chromaticity of a significant part of real colours of this system, presented in the *XYZ* colour space, have a negative value of the *y* coordinate. In Fig. 4, there are the chromaticity of radiation lying below the alychne (*x* axis) above the red and green dashed curves corresponding to the chromaticity diagrams of Yustova's system and the *LMS* system.

Table 5. The Matrix of Chromaticity Coordinates Transformation from *LMS* to *XYZ*

	<i>x</i>	<i>y</i>	<i>z</i>
<i>l</i>	0.738401448	0.261598552	0
<i>m</i>	1.326716353	-0.326716353	0
<i>s</i>	0.158619160	0.000000000	0.841380840

Table 6. Chromaticity Coordinates of the Primary Colours of a Trichromatic Physiological Colorimetric System (*LMS*)_{phys} in the *XYZ* System

	<i>x</i>	<i>y</i>	<i>z</i>
<i>l</i> _{phys}	0.734665727	0.265334273	0
<i>m</i> _{phys}	-5.942971132	6.942971132	0
<i>s</i> _{phys}	0.169147999	0.001096000	0.829756001

Let us discuss this matter in more detail as exemplified by *LMS*. Table 4 summarises the transition matrix from *LMS* to *XYZ* for a 2° field of view presented in [9].

This matrix allows transforming the tristimulus values obtained in *LMS* into the *XYZ* system. With that, the matching functions of all colours are positive in *LMS* and are identical to those in *XYZ* with an acceptable degree of accuracy after transformation by means of the matrix shown above.

Everything is fine here in terms of mathematical transformations since such colorimetric system may describe chromatic transformations. However, it does not correspond to a trichromatic physiological system. The question is: why?

The authors did not take one important circumstance into account: the obtained correspondence is not enough for a trichromatic physiological system. The sum of the positive tristimulus values of the initial *LMS* system shall also have a positive value of the *Y* coordinate after transformation into *XYZ* since only this coordinate estimates the total level of receptor excitement of a trichromat. This condition is not met for the *LMS* system.

If we divide the elements of the matrix in Table 4 by the sum of the tristimulus values (the last line of the table) and transpose it, we will obtain the matrix of transformation of the chromaticity coordinates from *LMS* into *XYZ*. This matrix of transformation of the chromaticity coordinates from *LMS* into *XYZ* is shown in Table 5.

In accordance with it, the reference colour triangle and the *LMS* chromaticity diagram are built in *XYZ*, Fig. 4.

As seen from the calculation results, the values of the *Y* coordinates of real colours are negative af-

ter transformation, which corresponds to a negative chromaticity impression. This is acceptable for a certain colorimetric system but is not acceptable for a physiological system, in which the receptor response level and perception of brightness cannot be negative.

In our opinion, a possible error in the authors' reasoning [9, 13] may be as follows: since a matrix of transition of the colour-matching functions of the new system into the *XYZ* system is obtained, using these transformed functions, we can also obtain a chromaticity diagram of the new system in *XYZ*. With such algorithm, the transformed chromaticity diagram of the new chromatic system is identical to the chromaticity diagram of the *XYZ* system. But it is completely wrong since the transformation matrices for tristimulus values and chromaticity coordinates are different matrices even though they are interrelated, and their application provides the results shown in Fig. 4.

The above said makes it necessary to keep studying definition of a trichromatic chromaticity coordinates using the results of this work and the results we obtained previously [14].

3. DEFINITION OF THE CHARACTERISTICS OF A TRICHROMATIC COLOUR SYSTEM (*LMS*)_{PHYS}

The results presented above allow us to draw the following conclusions which are necessary for further studies:

1. The tristimulus value of the reference colour stimulus *M*_{phys} of a trichromatic colour system in the *XYZ* colour space is positive.

Table 7. Colour-Matching Curves of $(LMS)_{phys}$

λ , nm	$\bar{l}_{phys}(\lambda)$	$\bar{m}_{phys}(\lambda)$	$\bar{s}_{phys}(\lambda)$	λ , nm	$\bar{l}_{phys}(\lambda)$	$\bar{m}_{phys}(\lambda)$	$\bar{s}_{phys}(\lambda)$
380	0.000047996	0.000015868	0.006450000	585	1.015781000	0.651020700	0.001400000
385	0.000077655	0.000027052	0.010550000	590	1.013966000	0.543377800	0.001100000
390	0.000142824	0.000052401	0.020050000	595	1.000195000	0.440604000	0.001000000
395	0.000250335	0.000101529	0.036210000	600	0.970421700	0.347934600	0.000800000
400	0.000448729	0.000187767	0.067850000	605	0.927106800	0.266067300	0.000600000
405	0.000695741	0.000326856	0.110200000	610	0.868016300	0.198163900	0.000340000
410	0.001230890	0.000691112	0.207400000	615	0.797102100	0.143856000	0.000240000
415	0.002050717	0.001390848	0.371300000	620	0.715048700	0.101833400	0.000190000
420	0.003311139	0.003017479	0.645600000	625	0.621535000	0.069788160	0.000100000
425	0.004864156	0.006833596	1.039050000	630	0.526486300	0.046393590	0.000050000
430	0.005938483	0.013006040	1.385600000	635	0.440730100	0.029934750	0.000030000
435	0.006199199	0.021856640	1.622960000	640	0.362020300	0.018612960	0.000020000
440	0.005965606	0.033090740	1.747060000	645	0.290185900	0.011099100	0.000010000
445	0.004945297	0.046378020	1.782600000	650	0.227190700	0.006482851	0.000000000
450	0.003315498	0.062865540	1.772110000	655	0.174772600	0.003675217	0.000000000
455	0.001378036	0.082967270	1.744100000	660	0.131505700	0.002030792	0.000000000
460	0.000000002	0.106395800	1.669200000	665	0.096523600	0.001134815	0.000000000
465	0.000679064	0.131758100	1.528100000	670	0.069528760	0.000610753	0.000000000
470	0.005627772	0.159636200	1.287640000	675	0.050550690	0.000323447	0.000000000
475	0.015087970	0.192102000	1.041900000	680	0.037142340	0.000152423	0.000000000
480	0.029070920	0.229571200	0.812950000	685	0.026107520	0.000053054	0.000000000
485	0.046369200	0.271288100	0.616200000	690	0.018005910	0.000016303	0.000000000
490	0.069482490	0.323555400	0.465180000	695	0.012561410	0.000003057	0.000000000
495	0.099112340	0.392096700	0.353300000	700	0.009006851	-0.000000638	0.000000000
500	0.136659300	0.479366600	0.272000000	705	0.006431389	-0.000000556	0.000000000
505	0.186263500	0.593108500	0.212300000	710	0.004591094	-0.000000190	0.000000000
510	0.246776300	0.718676500	0.158200000	715	0.003258209	-0.000000028	0.000000000
515	0.319084400	0.851824500	0.111700000	720	0.002298747	-0.000000017	0.000000000
520	0.396713400	0.974220200	0.078250000	725	0.001624736	-0.000000033	0.000000000
525	0.470518600	1.065551000	0.057250000	730	0.001141805	-0.000000106	0.000000000
530	0.541920700	1.132373000	0.042160000	735	0.000792863	-0.000000230	0.000000000
535	0.607344300	1.174839000	0.029840000	740	0.000547029	-0.000000285	0.000000000
540	0.667984800	1.196088000	0.020300000	745	0.000377488	0.000000121	0.000000000
545	0.724451900	1.197174000	0.013400000	750	0.000263307	0.000000132	0.000000000
550	0.777295000	1.179847000	0.008750000	755	0.000186408	0.000000178	0.000000000
555	0.827893600	1.146725000	0.005750000	760	0.000131654	0.000000066	0.000000000
560	0.875470800	1.097633000	0.003900000	765	0.000092642	-0.000000360	0.000000000
565	0.917929000	1.031844000	0.002750000	770	0.000065827	0.000000033	0.000000000
570	0.954915600	0.951865100	0.002100000	775	0.000046624	-0.000000434	0.000000000
575	0.984675400	0.859535600	0.001800000	780	0.000033216	-0.000000237	0.000000000
580	1.005857000	0.758198700	0.001650000	Σ	21.371419976	21.371248346	21.371540000

Table 8. Tristimulus Values of $(LMS)_{phys}$

λ , nm	$l_{phys}(\lambda)$	$m_{phys}(\lambda)$	$s_{phys}(\lambda)$	λ , nm	$l_{phys}(\lambda)$	$m_{phys}(\lambda)$	$s_{phys}(\lambda)$
380	0.00736825	0.002436069	0.990195681	585	0.608907784	0.390252989	0.000839227
385	0.007288365	0.002539014	0.990172621	590	0.650627248	0.34866692	0.000705832
390	0.007054711	0.002588293	0.990356996	595	0.693713201	0.305593221	0.000693578
395	0.006846883	0.002776904	0.990376213	600	0.73563815	0.263755402	0.000606448
400	0.00655208	0.002741657	0.990706263	605	0.77661829	0.222879103	0.000502608
405	0.00625539	0.002938757	0.990805853	610	0.813877037	0.185804169	0.000318794
410	0.005880366	0.00330167	0.990817964	615	0.846901518	0.152843487	0.000254994
415	0.00547235	0.003711486	0.990816164	620	0.875135377	0.124632086	0.000232538
420	0.00507899	0.004628542	0.990292468	625	0.898921292	0.100934079	0.000144629
425	0.004629233	0.006503555	0.988867212	630	0.918936696	0.080976034	8.72707E-05
430	0.004228049	0.00925997	0.986511981	635	0.936339329	0.063596935	6.37356E-05
435	0.003754779	0.013238298	0.983006923	640	0.951050045	0.048897414	5.25413E-05
440	0.003339987	0.018526643	0.97813337	645	0.963128827	0.036837983	3.31901E-05
445	0.002696567	0.025288964	0.972014469	650	0.972256805	0.027743195	0
450	0.001803576	0.034197817	0.963998607	655	0.979404528	0.020595472	0
455	0.000753665	0.045375856	0.953870479	660	0.984792232	0.015207768	0
460	1.08054E-09	0.059921182	0.940078816	665	0.988379752	0.011620248	0
465	0.000408942	0.079346673	0.920244384	670	0.991292318	0.008707682	0
470	0.003873465	0.109873882	0.886252653	675	0.993642206	0.006357794	0
475	0.01207917	0.153793565	0.834127265	680	0.995913019	0.004086981	0
480	0.027128718	0.214233752	0.758637531	685	0.997972	0.002028	0
485	0.049653411	0.290502735	0.659843854	690	0.999095382	0.000904618	0
490	0.080961363	0.377008454	0.542030183	695	0.999756704	0.000243296	0
495	0.117360899	0.464289524	0.418349577	700	1.000070813	-7.08127E-05	0
500	0.15389112	0.539811508	0.306297373	705	1.000086426	-8.64262E-05	0
505	0.187827729	0.598089388	0.214082882	710	1.000041445	-4.14445E-05	0
510	0.219619708	0.639589471	0.140790821	715	1.00000857	-8.57042E-06	0
515	0.248777628	0.66413425	0.087088122	720	1.000007528	-7.52828E-06	0
520	0.273749579	0.672254502	0.053995919	725	1.000020366	-2.03662E-05	0
525	0.295307106	0.668761622	0.035931272	730	1.000092501	-9.2501E-05	0
530	0.315721129	0.659716601	0.02456227	735	1.000289646	-0.000289646	0
535	0.335174664	0.648357557	0.016467779	740	1.000521212	-0.000521212	0
540	0.354486543	0.634740642	0.010772815	745	0.999678678	0.000321322	0
545	0.374388736	0.618686293	0.006924972	750	0.999499333	0.000500667	0
550	0.395390489	0.600158605	0.004450906	755	0.999045643	0.000954357	0
555	0.418050256	0.579046244	0.0029035	760	0.999499333	0.000500667	0
560	0.44282707	0.555200248	0.001972682	765	1.003902486	-0.003902486	0
565	0.470124552	0.528467014	0.001408434	770	0.999499333	0.000500667	0
570	0.500248968	0.498650911	0.001100121	775	1.009395936	-0.009395936	0
575	0.533407114	0.465617811	0.000975075	780	1.007199953	-0.007199953	0
580	0.569662883	0.429402646	0.000934471				

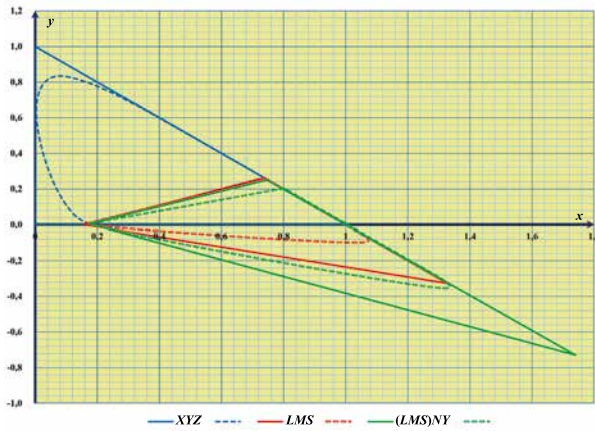


Fig. 5. RCS triangles and chromaticity diagrams of XYZ, LMS and Yustova (NY) in the XYZ system

2. The tristimulus values of the reference colours stimulus L_{phys} and M_{phys} of a trichromatic colour system in the XYZ colour space lie along the line $y = 1-x$.

The third necessary result obtained by the authors in [14] may be formulated as follows:

3. The reference colour stimulus M_{phys} of a trichromatic vision system is a real colour and is identical to the reference colour stimulus R of the CIE RGB colour system (1931), i.e. it has the chromaticity coordinates (0.7346657271, 0.2653342729, 0) in the XYZ system.

The method of determination of the characteristics of a trichromatic colour system $(LMS)_{phys}$ is based on determination of the chromaticity coordinates of its reference stimulus in the XYZ colour space. With that, the apexes of the reference colour L_{phys} and the section of the line along which the apex with the tristimulus value of the reference colour M_{phys} ($y=1-x$) lies are known in the coordinate triangle. The energy limitations of the installation at the boundaries of the spectral bands allowed conducting the studies of chromaticity differences only within the range of (420–660) nm, which led to a significant scatter of the tristimulus values of the reference colour S_{phys} when solving the system of equations (1), therefore, to define them, we will use the method described in the [15] by N.D. Nyberg. The nature of the method is that if we use the feature of the physiological system that the responses of its receptors cannot have negative values, the region of possible location of the S_{phys} apex of the reference-colour stimulus triangle in the XYZ system turns out to be very limited.

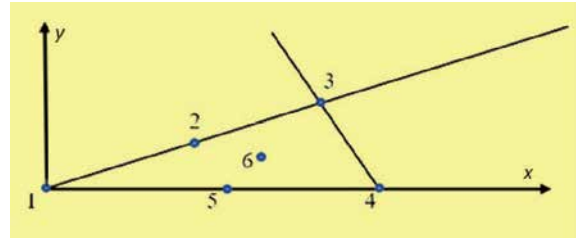


Fig. 6. The area of acceptable location of the apex S_{phys} of the coordinate triangle of a trichromatic colour system

First, if the coordinate y of the reference colour M_{phys} is positive, the coordinate triangle of a trichromatic colour system shall include the XYZ chromaticity diagram without intersecting it. Second, the value of the coordinate y of the reference colour S_{phys} cannot be below zero since otherwise the response to it will become negative. The area of possible location of the reference colour S_{phys} has the shape of a triangle and is shown in Fig. 5.

The line 1–5–4 in Fig. 5 is the alychne, i.e. it is the x axis in the XYZ system and the coordinate y becomes negative below it. The line 1–2–3 is continuation of the purple line of the chromaticity diagram of the XYZ system identical to the direction towards the triangle apex with the reference colour L_{phys} . The line 4–3 is a line parallel to the line $y = 1-x$ tangent to the XYZ chromaticity diagram to the left and corresponding to the boundary condition when the value of the coordinate Y of the reference colour M_{phys} changes its sign from $+\infty$ to $-\infty$. 6 options of the location of the reference colour S_{phys} (points 1 to 6 in Fig. 5) were considered. The points 1, 3, 4 are located at the apexes of the triangle, 2 and 5 are the midpoints of its side and 6 is its geometric centre. The results of calculations of the colour-matching functions of all these six points are presented in Fig. 6. The relative calculation errors of the chromaticity coordinates of an E-type equal-energy source when using any curves for the 6 considered locations of the reference colour S_{phys} are not much than seven decimal places.

The values closest to the average value of the dependences of colour-matching coefficients on wavelength are obtained when the reference colour S_{phys} is located in the centre of the triangle, i.e. in point 6 in Fig. 6. This allows us to calculate the tristimulus values of the reference colours of the physiological system $(LMS)_{phys}$, which are summarised in Table 6.

Using this matrix, we can get all the necessary parameters of $(LMS)_{phys}$, namely: colour-matching coefficients $\bar{l}_{phys}(\lambda)$, $\bar{m}_{phys}(\lambda)$, $\bar{s}_{phys}(\lambda)$, chromatici-

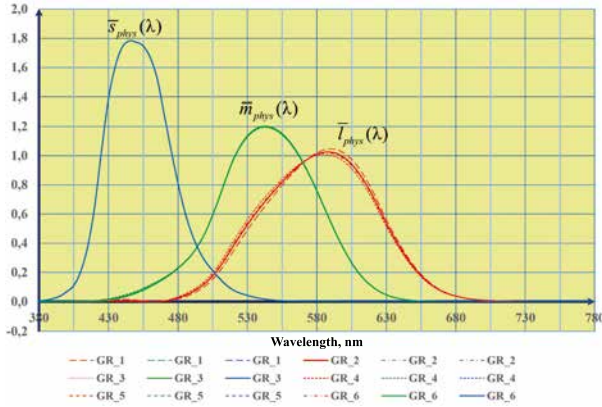


Fig. 7. Colour-matching coefficients of a trichromatic physiological system obtained based on the data from Guild and Wright [3, 4] for 6 possible positions of the reference colour S_{phys}

ty coordinates and diagram $l_{phys}(\lambda)$, $m_{phys}(\lambda)$, luminance factors of the system, matrices for transformation from $(LMS)_{phys}$ to the standard colorimetric systems XYZ , RGB (1931) [16] and vice versa, as well as the tristimulus values of its reference colours in these systems. These parameters are shown below:

– Fig. 7 and Table 7 present the colour-matching coefficients of $(LMS)_{phys}$;

– Fig. 8 and Table 8 present the chromaticity diagram and chromaticity coordinates of $(LMS)_{phys}$;

– The expressions (3) to (6) and the tristimulus values of the reference colours stimulus, presented in Table 9, describe the interrelation between $(LMS)_{phys}$ and the standard CIE colorimetric systems.

$$\begin{pmatrix} \bar{l}_{phys}(\lambda) \\ \bar{m}_{phys}(\lambda) \\ \bar{s}_{phys}(\lambda) \end{pmatrix} = \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \times \begin{pmatrix} 0.605698611 & -0.507865231 & 0.000000000 \\ 0.518459503 & 1.406192931 & 0.000000000 \\ -0.124158114 & 0.101672300 & 1.000000000 \end{pmatrix}; \quad (3)$$

$$\begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} = \begin{pmatrix} \bar{l}_{phys}(\lambda) \\ \bar{m}_{phys}(\lambda) \\ \bar{s}_{phys}(\lambda) \end{pmatrix} \times \begin{pmatrix} 1.261118115 & 0.455469536 & 0.000000000 \\ -0.464970813 & 0.543209593 & 0.000000000 \\ 0.203852698 & 0.001320870 & 1.000000000 \end{pmatrix}; \quad (4)$$

$$\begin{pmatrix} \bar{l}_{phys}(\lambda) \\ \bar{m}_{phys}(\lambda) \\ \bar{s}_{phys}(\lambda) \end{pmatrix} = \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} \times \begin{pmatrix} 2.195536519 & 0.000000000 & 0.000000000 \\ 3.434036778 & 5.571394241 & 0.056506753 \\ 0.021101958 & 0.079281015 & 5.594168503 \end{pmatrix}; \quad (5)$$

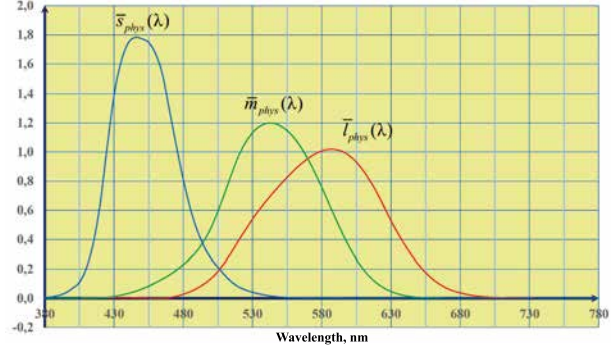


Fig. 8. Colour-matching coefficients of $(LMS)_{phys}$

$$\begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} \bar{l}_{phys}(\lambda) \\ \bar{m}_{phys}(\lambda) \\ \bar{s}_{phys}(\lambda) \end{pmatrix} \times \begin{pmatrix} 0.455469536 & 0.000000000 & 0.000000000 \\ -0.280760403 & 0.179514088 & -0.001813274 \\ 0.002260867 & -0.002544088 & 0.178783274 \end{pmatrix}. \quad (6)$$

Table 10 presents the luminance factors β of the system $(LMS)_{phys}$. The difference of the spectral luminous efficiency $V(\lambda)$ obtained for these luminance factors and colour-matching functions from the standard CIE photometric observer is not higher than seven decimal points.

Therefore, all parameters of a trichromatic physiological system $(LMS)_{phys}$ necessary for their application for colorimetric calculations are defined.

4. APPLICATION OF THE OBTAINED RESULTS FOR DEFINITION OF COLOUR THRESHOLDS

The expression (1) allows us to get the results similar to MacAdam experimental results [19] used for evaluation of colour thresholds. The results of the calculations and the experimental MacAdam ellipses multiplied by 10 times are shown in Fig. 9.

The object and the background with different chromaticity and luminance constants were created using three monochromatic sources:

For sources with $\lambda = 380, 520, 700$ nm;

For sources with $\lambda = 440, 505, 640$ nm;

For sources with $\lambda = 440, 535, 640$ nm;

For MacAdam ellipses.

After calculating the colour thresholds in the $(LMS)_{phys}$ and transforming them into the XYZ system, it was found that the uniform-chromaticity curves (with shape close to elliptical) depend not only on the chromaticity coordinates but also on the spectral range of radiation that provides these colours. With changes of radiation spectrum, not only

Table 9. Chromaticity Coordinates of Reference Colours when Transforming from Standard CIE Colorimetric Systems to $(LMS)_{phys}$

XYZ B $(LMS)_{phys}$				
	l_{phys}	m_{phys}	s_{phys}	Σ
x	6.191124241	-5.191124241	0.000000000	1.000000000
y	0.269378249	0.730621751	0.000000000	1.000000000
z	-0.127014130	0.104011073	1.023003057	1.000000000
Σ	6.333488360	-4.356491417	1.023003057	
$(LMS)_{phys}$ B XYZ				
	x	y	z	Σ
l_{phys}	0.734665727	0.265334273	0	1.000000000
m_{phys}	-5.942971132	6.942971132	0	1.000000000
s_{phys}	0.169147999	0.001096000	0.829756001	1.000000000
Σ	-5.039157405	7.209401405	0.829756001	

RGB B $(LMS)_{phys}$				
	l_{phys}	m_{phys}	s_{phys}	Σ
r	0.388544098	0.607721490	0.003734412	1.000000000
g	0.000000000	0.985969641	0.014030359	1.000000000
b	0.000000000	0.010000002	0.989999998	1.000000000
Σ	0.388544098	1.603691133	1.007764769	
$(LMS)_{phys}$ B RGB				
	r	g	b	Σ
l_{phys}	1.000000000	0.000000000	0.000000000	1.000000000
m_{phys}	2.724253101	-1.741847521	0.017594420	1.000000000
s_{phys}	0.012665918	-0.014252592	1.001586674	1.000000000
Σ	3.736919019	-1.756100113	1.019181094	

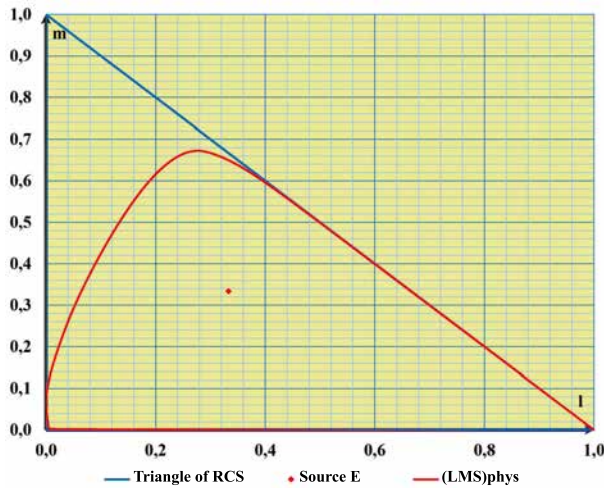


Fig. 9. The reference colour triangle and the chromaticity diagram of $(LMS)_{phys}$

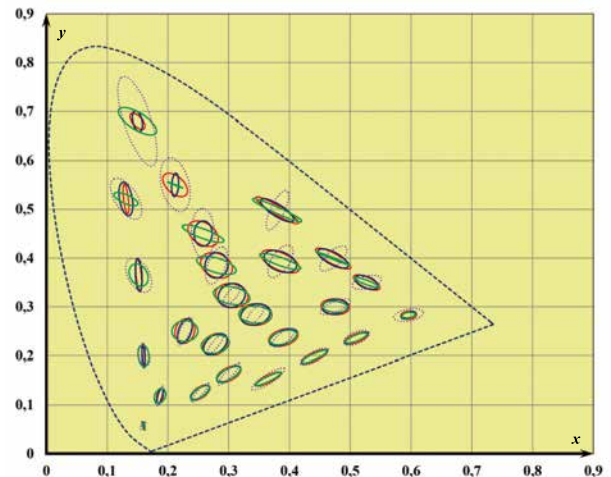


Fig. 10. The results of calculation of colour thresholds and experimental MacAdam ellipses multiplied by 10 times

the size of the ellipses but also their orientation in the colour space changes.

5. DISCUSSION OF THE RESULTS

1. Comparison of the results obtained with the materials published in [9] and adopted by CIE in 2006 [18] shows that the colour-matching coefficients of the M and S colours are virtually identical to M_{phys} and S_{phys} both in terms of shape and positions of the maximums. Only the colour-matching coefficients of the L and L_{phys} colours differ significantly from each other not only in terms of curve shapes (Fig. 10, a) but also in terms of positions of their maximums (Fig. 10, b). The maximum of the colour-matching coefficients of the L colour is at wavelength of 565 nm whereas the maximum

of the M_{phys} colour in $(LMS)_{phys}$ is at wavelength of 585 nm, i.e. it is displaced to the red region of the spectrum by 20 nm. This will lead to significant differences in the results of calculation in the systems $CIE2006$ and $(LMS)_{phys}$ for the sources emitting predominantly in the red region of the spectrum.

2. An indirect proof of correctness of the results obtained in the work is the fact that all experimental studies of colour-matching coefficients of trichromatic vision conducted using real reference colours [3, 4, 5, 6] had the chromaticity coordinates of the reference colours R (red region) and B (blue region) in the XYZ system close to the chromaticity coordinates of M_{phys} and S_{phys} $(LMS)_{phys}$, though not identical to them. The tristimulus value of the reference colour G (green region of the spectrum) was always positive (since real colours were used). The maxi-

Table 10. Luminance Factors of the System $(LMS)_{phys}$

β_l	β_m	β_s	Σ
0.455469525	0.543209604	0.001320871	1.000000000

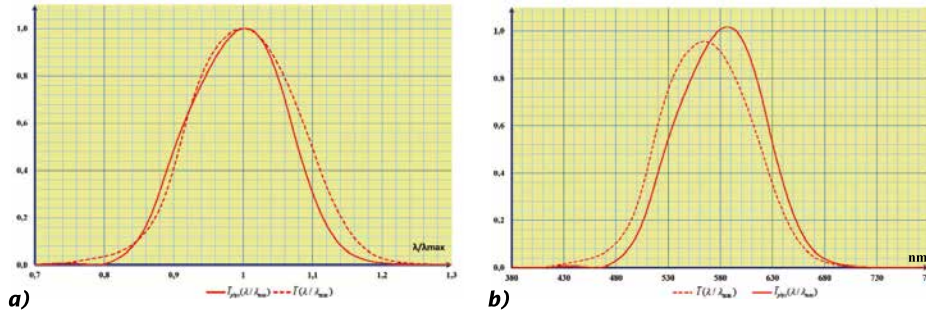


Fig. 11. The dependences of colour-matching functions $\bar{T}_{phys}(\lambda/\lambda_{max})$ and $\bar{T}(\lambda/\lambda_{max})$ per unit (a) and dependences $\bar{T}_{phys}(\lambda)$ and $\bar{T}(\lambda)$ in absolute terms (b)

mums of the matching functions for reference colours R were located at about 600 nm in all the experiments. This value is much closer to the one we obtained (585 nm) than the positions of the maximums of the M reference stimulus colour-matching functions obtained when using dichromate (555 nm, 565 nm, 570 nm) [9, 13].

3. The negative values of tristimulus value Y of real colours obtained in the $CIE2006$ colorimetric system after being transformed into the XYZ system show that even though this system is colorimetric, it is not a physiological trichromatic system for which the XYZ system is built.

4. Calculation of the colour thresholds showed that in the $(LMS)_{phys}$ and in the XYZ system, these thresholds depend not only on chromaticity of radiations and direction of chromaticity change in a colour space (since these systems are not uniform-chromaticity) but also on spectra of the radiation forming these chromaticity. This factor had not been accounted for in MacAdam experiments, and probably this was the cause of him obtaining repeatable results in the form of ellipses only for one of the studied observer.

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